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Performance of a Range-Ambiguous MTI and Doppler Filter System

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In this report the clutter-rejection performance of a range-ambiguous MTI and doppler-filter system is analyzed. The reduction of correlation time in a range-ambiguous system in general improves the clutter-rejection performance. However, due to the fold over phenomenon, a faraway target has to compete with the near-in clutter; hence a higher improvement factor is required. Examples presented here which take into account these effects show that the improvement factor of a range-ambiguous MTI system is slightly better than that of a range-unambiguous system. However the performance of a doppler-filter processor in a range-ambiguous system is worse.

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CONTENTS

PERFORMANCE OF A RANGE-AMBIGUOUS MTI AND DOPPLER FILTER SYSTEM

INTRODUCTION

In a radar system, in order to reject clutter noise, a sequence of pulses is often required. A proper filter can then be formed which rejects the stationary clutter and passes through the nonstationary targets. The performance of such a clutter-rejection filter depends on the sampling time (or the interpulse time). Often the radar system is required to measure the target doppler frequency. The resolution with which the doppler frequency can be measured is also determined by the sampling time (or the interpulse time). In either case, the faster the sampling rate, the better the performance possible.

However the radar unambiguous range is determined by the interpulse time. At a higher sampling rate some far-range intervals fold over and the target range becomes ambiguous. Furthermore clutter at faraway range cells folds over on the near-in range cells. Thus the clutter level at a certain range cell is actually increased. Moreover a faraway target has to compete with the near-in clutter; thus a range-ambiguous system requires a higher clutter-rejection capability to achieve the same performance it would have if it were a range-unambiguous system. In this report the performance of such a radar system is analyzed and some typical examples are presented.

CORRELATION FUNCTION

A radar transmits a series of identical pulses with an interpulse time of T. The unambiguous range of this radar is

$$r_0 = \frac{Tc}{2}, \tag{1}$$

where c is the velocity of light. Assume that the radar detection range r_a is many times larger than r_0. For convenience it is assumed that the radar echoes from targets or clutter at ranges greater than r_a are so small that they are negligible. Let N be an integer number (called the foldover index) such that

$$N = r_a/r_0. \tag{2}$$

The received radar signal is range gated. In what follows we shall limit ourselves to examine only the signal at a certain range bin. The kth received pulse at a range r (which immediately follows the kth transmitted pulse) is the sum of the return of the kth

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transmitted pulse at range r, the $(k-1)$th pulse at range $r + r_0$, the $(k-2)$th pulse at range $r + 2r_0$, etc. Hence

$$S_k(r) = \sum_{i=0}^{k-1} P_{k-i}(r + ir_0). \tag{3}$$

The correlation of two returns at times T_k and T_ℓ (where $T_k = kT$) is

$$R(T_k, T_\ell, r) = \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} \overline{P_{k-i}(r + ir_0)P_{\ell-j}(r + jr_0)}. \tag{4}$$

Because the radar returns from two range cells are statistically independent, one has

$$\overline{P_{k-i}P_{\ell-j}} = 0, \qquad \text{if } i \neq j.$$

Hence the double summation of Eq. (4) becomes

$$R(T_k, T_\ell, r) = \sum_{i=0}^{k-1} P_{k-i}(r + ir_0)P_{\ell-i}(r + ir_0). \tag{5}$$

Three cases are of interest. Let the radar have a K-pulse canceler (or K-point doppler filter). The MTI (or doppler-filter) pulses can be labeled as $k = 0, 1, \ldots, K-1$. In the first case we assume that identical pulses are transmitted continuously. Pulses prior to the first MTI (or doppler-filter) pulse are the same as the MTI pulses. This can happen when extra pulses are transmitted prior to the first usable MTI pulses or, in a scanning radar, when the pulses transmitted are not changed from sweep to sweep. Although the clutter cell will be slightly different from sweep to sweep due to the rotation of the antenna beam, this effect is assumed to be negligibly small and is ignored. Under this condition Eq. (5) holds.

In the second case it is assumed that no pulse is transmitted prior to the first MTI pulse. This case occurs when the antenna beam of a phased array is steered to a different direction and no previously radiated energy is available in this direction. The first transmitted pulse in that direction is used as the first MTI pulse. The returns of the first, second, ..., and $(K-1)$th pulses are

$$S_0 = P_0(r),$$

$$S_1 = P_1(r) + P_0(r + r_0),$$

$$\ldots,$$

$$S_{K-1} = P_{K-1}(r) + P_{K-2}(r + r_0) + \ldots + P_0[r + (K-1)r_0], \tag{6}$$

where $S_0, S_1, \ldots, S_{K-1}$ are the received pulses and $P_i(r + jr_0)$ represents the return of the ith transmitted pulse from a range cell $r + jr_0$.

An MTI system performs cancellation based on the principle that the stationary clutter return does not vary from pulse to pulse. Hence a sequence of identical pulses must be received to achieve cancellation. However, in this case of no pulse transmitted prior to the MTI pulse, one sees from Eqs. (6) that the received pulses vary from pulse to pulse; thus their capability of canceling the stationary clutter will be greatly reduced. One may argue that this variation may be compensated by suitably choosing a set of filter weights. But this variation depends on whether a range cell contains clutter or not, and this information is not known a priori.

A doppler-filter system also must receive a sequence of identical pulses to achieve integration gain. For clutter rejection the receiving pulses are usually weighted to reduce sidelobes and to improve the clutter rejection. Therefore variation of the received pulses is again not desirable.

To avoid this problem of variations of the clutter returns, a few extra pulses can be used until the received pulses are stabilized; that is, the second case can be changed to the first case.

In the third case it is assumed that the radar is operating in a frequency-agility mode; that is, the radar pulses transmitted prior to the first of the MTI pulse group are at a different frequency. The returns can be represented as

$$\begin{aligned}
S_0 &= P_0(r) + P'_{-1}(r + r_0) + P'_{-2}(r + 2r_0) + \dots,\\
S_1 &= P_1(r) + P_0(r + r_0) + P'_{-1}(r + 2r_0) + \dots,\\
S_{K-1} &= P_{K-1}(r) + P_{K-2}(r + r_0) + P_{K-3}(r + 2r_0) + \dots. \qquad (7)
\end{aligned}$$

The returns P'_{-1}, P'_{-2}, ... are from the previously transmitted pulses, which have a frequency different from that of the current pulses.

During the receiving time the local oscillator is set at a frequency compatible with the current pulses, so that the primed pulses from the mixer are noncoherent and uncorrelated from pulse to pulse. They act similar to random noise, if they are not rejected by the band-limit filter. This type of wind-band noise cannot be filtered out by an MTI filter. Its presence affects the signal-to-noise ratio and makes detection more difficult. However it has little effect on the MTI performance. In the subsequent discussion we shall ignore its presence. If the presence of the primed pulses is ignored, this case becomes identical to the second case. Conclusions drawn from there can be directly applied to the present case.

For simplification in the subsequent discussion, we shall assume that the pulses prior to the first MTI pulses are identical to the MTI pulses.

IMPROVEMENT FACTOR

By use of Van Trees' model [1] the correlation function can be represented

$$R(T_k, T_\ell, r) = \sum_i \frac{C_i}{(r + ir_0)^L} \int_{-\infty}^{\infty} G_i(f) e^{j2\pi fT(k-\ell)}\, df, \tag{8}$$

where $G_i(f)$ is the clutter spectrum density function at the range cell $r + ir_0$, C_i is a constant which is a function of radar power, antenna gain, and the clutter radar cross section, and L equals 2 for volume clutter and equals 3 for surface clutter. In this formulation all radar pulses are assumed identical.

If a target resides at a range $r + mr_0$ and has a spectral density function $H(f)$, its correlation function is

$$R_t(T_k, T_\ell, r) = \frac{C_t}{(r + mr_0)^4} \int H(f) e^{j2\pi fT(k-\ell)}\, df, \tag{9}$$

where C_t is a constant which is a function of the pulse power, radar parameters, and target cross section.

For a range-unambiguous MTI system the clutter correlation function at a range cell $r + mr_0$ is

$$R_0(T_k, T_\ell, mr_0) = \frac{C_m}{(r + mr_0)^L} \int G_m(f) e^{j2\pi fT_N(k-\ell)}\, df, \tag{10}$$

where $T_N = NT$. The target correlation function remains the same as that of Eq. (9), except the interpulse time is T_N instead of T.

The signal-to-clutter ratio at the output of the filter is

$$(\mathrm{SCR})_o = \frac{\sum_k \sum_\ell a_\ell a_k^* R_t(T_k, T_\ell, r)}{\sum_k \sum_\ell a_\ell a_k^* R(T_k, T_\ell, r)} \tag{11}$$

where a_k is the filter weight and a_k^* is the conjugate of a_k.

For convenience in making comparisons we shall assume that the clutter at the input of the filter is the same as that of a range-unambiguous MTI system. For a worst case we shall consider a target at the farthest range interval that

$$r_T = r + (N - 1)r_0. \tag{12a}$$

The input signal-to-clutter ratio is then

$$(\mathrm{SCR})_i = \frac{C_T}{r_T^4} \Bigg/ \frac{C_N}{r_T^L} . \tag{12b}$$

For convenience assume that the clutter is uniform throughout the entire range interval. Inserting Eqs. (8) and (9) into Eq. (11), one finds the improvement factor of a range-ambiguous system is

$$I_a = \frac{(\mathrm{SCR})_o}{(\mathrm{SCR})_i}$$

$$= \frac{\sum_k \sum_\ell a_k a_\ell^* \int H(f) e^{j2\pi f T(k-\ell)}\, df}{\left[\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f T(k-\ell)}\, df\right] A_L(r_0, r, N)} , \tag{13a}$$

where

$$A_L(r_0, r, N) = \sum_{i=0}^{N-1} \left[\frac{(r/r_0) + (N-1)}{(r/r_0) + i}\right] \tag{13b}$$

For the range-unambiguous case, from Eq. (9) one finds

$$I_u = \frac{\sum_k \sum_\ell a_k a_\ell^* \int H(f) e^{j2\pi f N T(k-\ell)}\, df}{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f N T(k-\ell)}\, df} . \tag{14}$$

Two cases are of interest. In the first case the target spectrum density is assumed to be an impulse function:

$$H(f) = \delta(f - f_t).$$

Under this condition the target is assumed to have a constant doppler frequency f_t. Its correlation function then becomes

$$\int \delta(f - f_t) e^{j2\pi f T(k-\ell)}\, df = e^{j2\pi f_t T(k-\ell)},$$

and the improvement factors for range-ambiguous and range-unambiguous systems are

$$I_a = \frac{\left|\sum_{\ell} a_\ell\right|^2}{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f T(k-\ell)}\, df\, A_L(r_0, r, N)} \tag{15}$$

and

$$I_u = \frac{\left|\sum_{\ell} a_\ell\right|^2}{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f N T(k-\ell)}\, df} \tag{16}$$

This corresponds to a doppler-filter case.

In the second case of interest $H(f)$ is assumed to be a constant value of unity; then

$$\begin{aligned} \int H(f) e^{j2\pi f T(k-\ell)}\, df &= 1, \qquad k = \ell, \\ &= 0, \qquad k \neq \ell. \end{aligned}$$

This corresponds to the case for an MTI in which the target doppler is not known a priori. One may assume that it has a uniform distribution; therefore

$$I_a = \frac{\sum_{\ell} \left| a_\ell\right|^2}{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f T(k-\ell)}\, df\, A_L(r_0, r, N)} \tag{17}$$

and

$$I_u = \frac{\sum_{\ell} \left| a_\ell\right|^2}{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f N T(k-\ell)}\, df} . \tag{18}$$

One of the most important questions is whether by use of a range-ambiguous MTI (or doppler filter) the clutter-rejection performance can be improved. This can be easily answered by comparing the improvement factors of these two types of clutter-rejection systems. One has

$$\frac{I_a}{I_u} = \frac{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f T(k-\ell)}\, df}{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f NT(k-\ell)}\, df} A_L(r_0, r, N) \tag{19}$$

For the case when the MTI pulses are identical to pulses transmitted prior to MTI pulses, the factor $A_L(r_0, r, N)$ can be factored out. The above ratio can be examined in two parts. The first part of this ratio,

$$\frac{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f T(k-\ell)}\, df}{\sum_k \sum_\ell a_k a_\ell^* \int G(f) e^{j2\pi f NT(k-\ell)}\, df},$$

is the ratio of the clutter residues of the range-ambiguous MTI (or doppler filter) system to that of the range-unambiguous system. This ratio is a function of the correlation time. It is well known that the minimum (optimum) clutter output of an MTI system is proportional to the correlation time [2]. As the correlation time increases, the clutter output also increases. For a range-ambiguous system the correlation time is T, and for a range-unambiguous system the correlation time is NT. Thus for the same type of clutter a range-ambiguous system produces less clutter residues and has a better improvement factor. As the foldover index N increases, the correlation time of a range-ambiguous system becomes shorter, and this leads to a better improvement factor.

The second factor, $A_L(r_0, r, N)$, is a function of the residue range r (Eq. (12a)) and the foldover index N. When $r = r_0$ this factor becomes

$$A_L(r_0, r_0, N) = \sum_{i=1}^{N} \left(\frac{N}{i}\right)^L.$$

Hence one may manipulate $A_L(r_0, r, N)$ into the form

$$A_L(r_0, r, N) = \sum_{i=1}^{N} \left(\frac{N}{i}\right)^L \frac{\sum_{i=0}^{N-1} \left[\frac{(r/r_0) + (N-1)}{(r/r_0) + i}\right]^L}{\sum_{i=1}^{N} \left(\frac{N}{i}\right)^L}.$$

This factor can be approximated as

$$A_L(r_0, r, N) \approx \sum_{i=1}^{N} \left(\frac{N}{i}\right)^L \left(\frac{r_0}{r}\right)^L.$$

The factor A_L thus consists of two parts; the first part is due to the effect of the foldover index N, and the second part is due to the residue range r.

As the foldover index N increases, the number of foldover clutter cells increases, and the amount of clutter in each range cell increases too. Since a target signal is competing with this clutter level, a better improvement factor is required to achieve the same visibility. Thus the effect of the foldover index N on the one hand is to reduce the MTI correlation time and to achieve better clutter rejection. The effect of N on the other hand is to increase the number of foldover clutter cells; hence the amount of clutter which must be rejected must be increased.

When the residue range r is small, the target signal competes with clutter close to the radar. The amount of clutter required to be rejected is large. The amount of increased clutter due to this effect is shown in Fig. 1. The ratio

$$\sum_{i=0}^{N-1} \left[\frac{(r/r_0) + (N-1)}{(r/r_0) + i}\right]^L \Big/ \sum_{i=1}^{N} \left(\frac{N}{i}\right)^L \equiv \text{clutter ratio} \tag{20}$$

is plotted as a function of the ratio of the residue range and the unambiguous range (r/r_0) for the cases of surface clutter and volume clutter. Since the worst case actually occurs when r is equal to the radar minimum range r_{min} (since targets are not detectable when $r < r_{min}$), these plots thus also show the effect of radar minimum range. If the radar minimum range were equal to r_0, the increased clutter ratio would be at 0 dB. When the radar minimum range reduces, this clutter ratio increases, and is approximately a function of $(r/r_0)^L$.

EXAMPLES

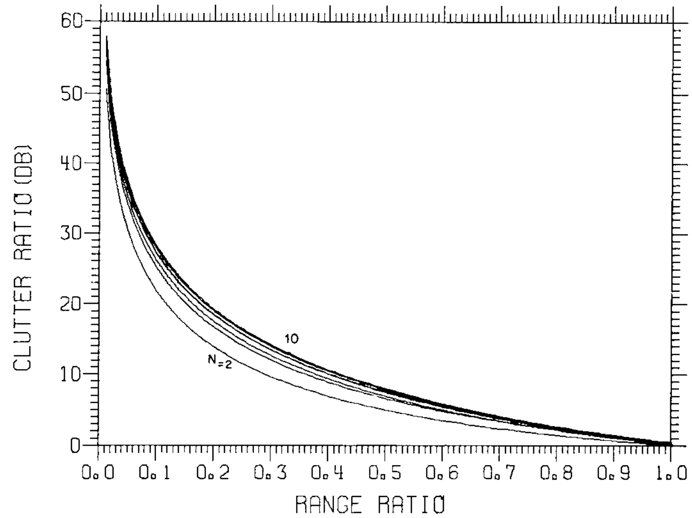
In the following examples the clutter spectrum density is assumed Gaussian, having the form

$$G(f) = \frac{1}{\sqrt{2\pi\sigma}} e^{-f^2/2\sigma^2}, \tag{20}$$

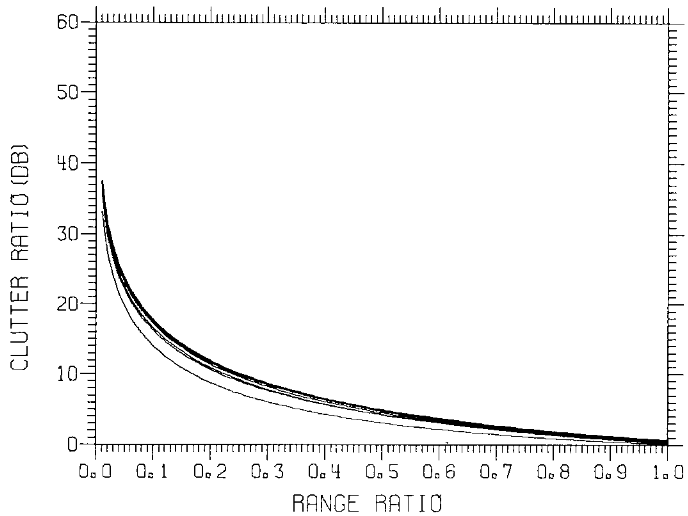
where σ is the standard deviation, with the correlation function being

$$\int_{-\infty}^{\infty} G(f) e^{j2\pi f(i-j)T}\, df = e^{-2\pi^2\sigma^2(i-j)^2T^2}. \tag{21}$$

In the plots σ is normalized with respect to PRF (which is the reciprocal of the required interpulse time T for the equivalent range-unambiguous case) for convenience in making comparisons.



(a) Surface clutter (L = 3)



(b) Volume clutter (L = 2)

Fig. 1 — Increase in clutter level due to close-in clutter as the residue range decreases, shown by plotting the clutter ratio, given by Eq. (20), as a function of the range ratio r/r_0. This plot also shows the effect of radar minimum range r_{min}, if the range ratio is interpreted as r_{min}/r_0.

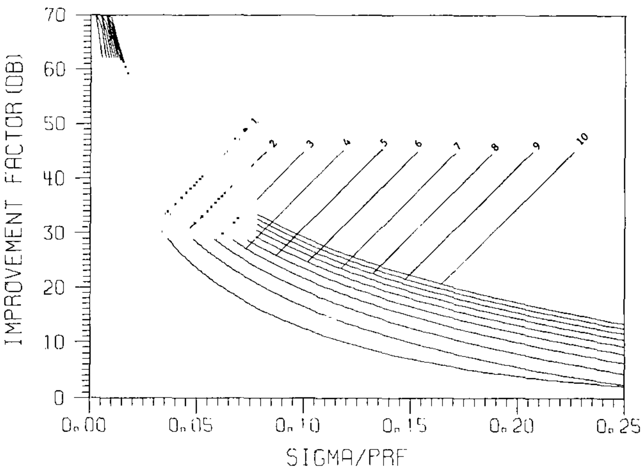


Fig. 2 — Improvement factor of surface clutter of a three-pulse canceler

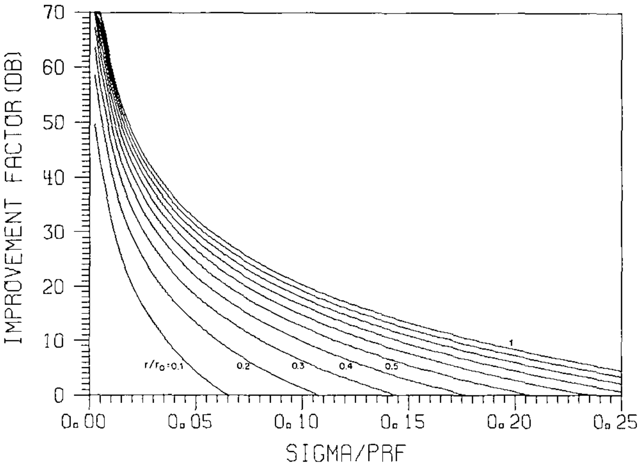


Fig. 3 — Improvement factor of surface clutter of the three-pulse canceler of Fig. 2 when N = 10 and the minimum-range effect of Fig. 1a is taken into account

Figure 2 shows the improvement factor of a three-pulse canceler as a function of the normalized σ. The filter weights are assumed to be binomial, and surface clutter is assumed. When the foldover index N = 1, a range-unambiguous case is represented. For example, if it is required to design a radar to cover a range of 150 km for a range-unambiguous case, the radar PRF must be 1 kHz. Assume that the clutter spectrum density function has a standard deviation of 100 Hz. Then the normalized σ is 0.1. Now if a range-ambiguous MTI system is used, the radar PRF could become for example 2 KHz (N = 2). The actual normalized σ would be 0.05. However for the convenience of comparison the improvement factor of both cases is plotted at the same abscissa (at σ = 0.1). Figure 2 shows that the improvement factor of these two cases is respectively 12 dB and 14 dB. Thus, given the clutter spectral standard deviation and the required radar range, curves in Fig. 2 represent the improvement factor that can be achieved either by use of

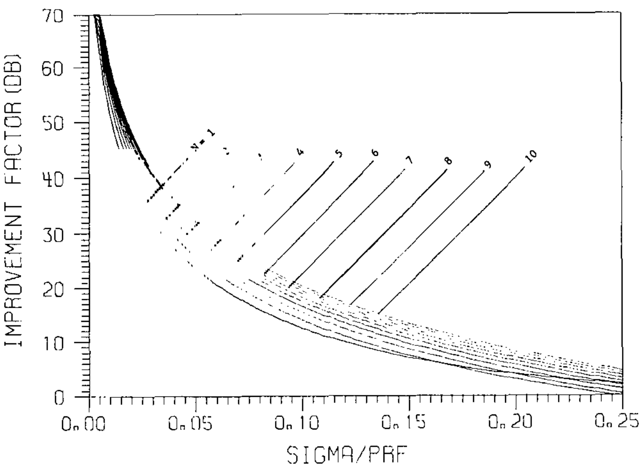


Fig. 4 — Improvement factor of volume clutter of the three-pulse canceler of Fig. 2

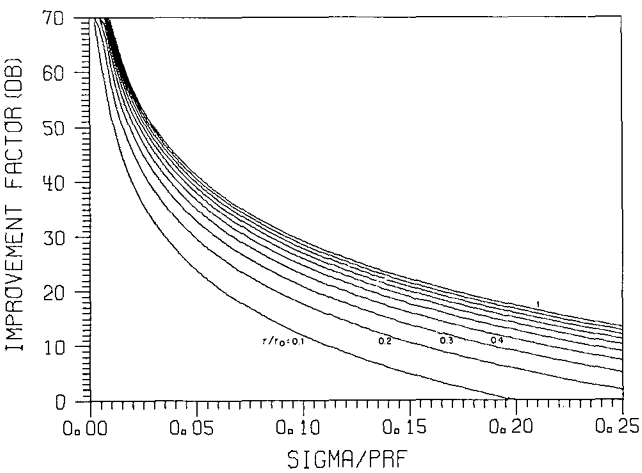
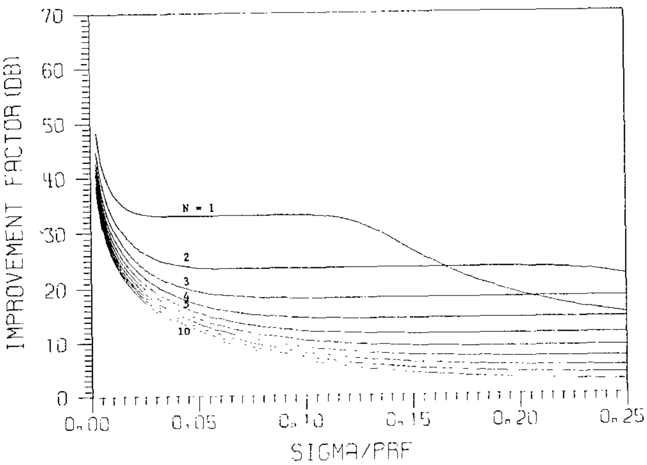


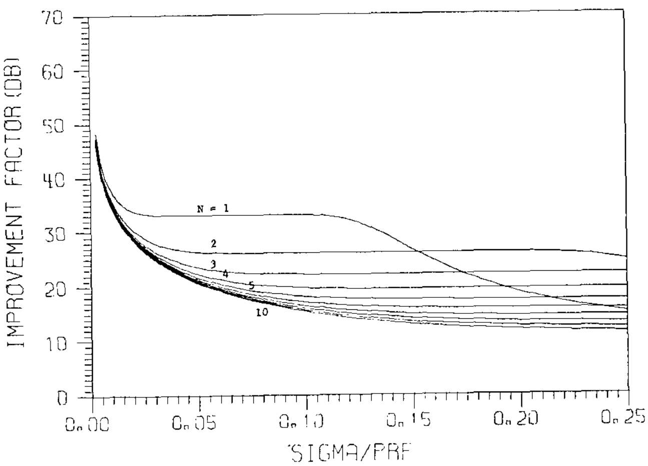
Fig. 5 — Improvement factor of volume clutter of the three-pulse canceler of Fig. 4 when N = 10 and the minimum-range effect is taken into account

range-ambiguous, or range-unambiguous MTI systems. The figure shows that the improvement obtained by use of the range-ambiguous system is not much greater than that of a range-unambiguous system.

The curves in Fig. 2 are for the case in which the minimum-range effect of Fig. 1 is ignored. In other words, only the part $\Sigma_{i=1}^{N}(N/i)^L$ of the foldover-index effect is taken into account. Therefore the actual improvement factor that can be achieved would be the improvement factor (in dB) shown in Fig. 2 minus the dB value of clutter ratio due to the minimum-range effect shown in Fig. 1. Figure 3 shows this improvement factor that takes into account both the minimum-range and foldover effects for a three-pulse canceler having a foldover index N = 10.



(a) Surface clutter



(b) Volume clutter

Fig. 6 — Improvement factor of a 16-point FFT doppler filter

Figure 4 shows the performance of the three-pulse canceler of Fig. 2 against volume clutter. Since $L = 2$ for volume clutter and $L = 3$ for surface clutter, the factor $\Sigma_i (N/i)^L$ is smaller for volume clutter. Therefore the improvement factor of a range-ambiguous MTI against volume clutter is better. Figure 5 shows the improvement of this same three-pulse canceler when $N = 10$ and the minimum-range effect is taken into account.

Figure 6 shows the improvement factor of a 16-point FFT doppler filter against surface and volume clutter, without taking the minimum-range effect into account. The improvement factor is examined at a filter corresponding to a doppler of 8/16 of the PRF. The curve marked $N = 1$ represents the improvement factor of a range-unambiguous system. The improvement factors of this curve can be divided into three regions. For a normalized standard deviation (σ/PRF) less than 0.02 the clutter is contained in a sidelobe null; hence it is better filtered and gives a high improvement factor. For values of normalized σ from 0.02 to 0.12 the clutter is confined in the sidelobe region. Since the weights of the doppler filter have a Chevyshev distribution with a 30-dB sidelobe design, the improvement factor is limited to this level of approximately 30 dB (33 dB according to Fig. 6). In the region where the clutter normalized standard deviation is greater than 0.12, part of the clutter gets into the main beam region, and the improvement factor falls off rapidly. For a corresponding range-ambiguous system, say for $N = 2$, in a range of σ/PRF from 0 to 0.04 the improvement factor is good. However due to the foldover clutter additional cancellation is required, and the net improvement factor is actually worse than that of a range-unambiguous system. For values of σ/PRF from 0.04 to 0.24 the clutter is in a sidelobe region, and the improvement factor stays fairly constant, as is the case for a range-unambiguous system. However the net improvement factor is worse due to the clutter foldover. The improvement factor for the range-ambiguous system is slightly better for volume clutter (Fig. 6b) than for surface clutter (Fig. 6a), since the foldover effect of the volume clutter is not as strong as it is for surface clutter.

Figure 7 shows a case of a three-pulse canceler when pulses transmitted prior to the first MTI pulse are either at a different frequency or do not exist. As pointed out earlier, the performance of such a range-ambiguous system will be degraded. Figure 8 shows this degradation for a 16-point FFT doppler filter.

RANGE-AMBIGUITY RESOLUTION

One difficulty encountered with a range-ambiguous system is the determination of a target's actual range. Since a target in the ranges of r, $r + r_0$, $r + 2r_0$, ..., $r + (N - 1)r_0$ falls into the same range bin r, it is difficult to resolve this ambiguity. One way to resolve it is to use several different PRFs. Assume that the number of range bins provided by these PRFs are r_1, r_2, ..., r_k, and assume that these numbers are prime to each other. Then a target at a range greater than the unambiguous range will fall into different range bins with the different PRFs. Therefore a total of

$$R = \prod_{j=1}^{k} r_j$$

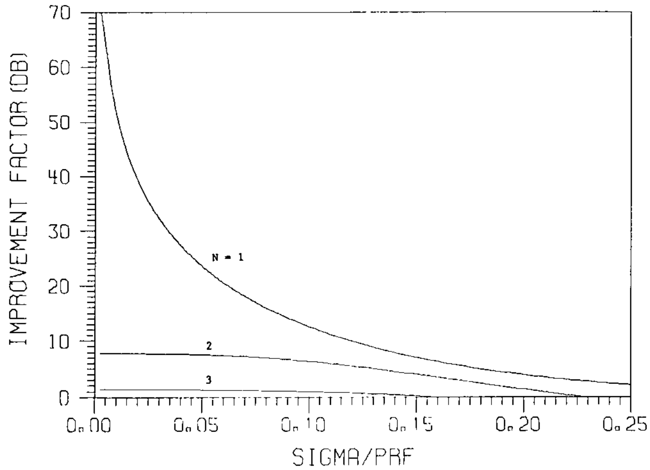


Fig. 7 -- Improvement factor of surface clutter of the three-pulse canceler when pulses transmitted prior to the first MTI pulse are at a different frequency

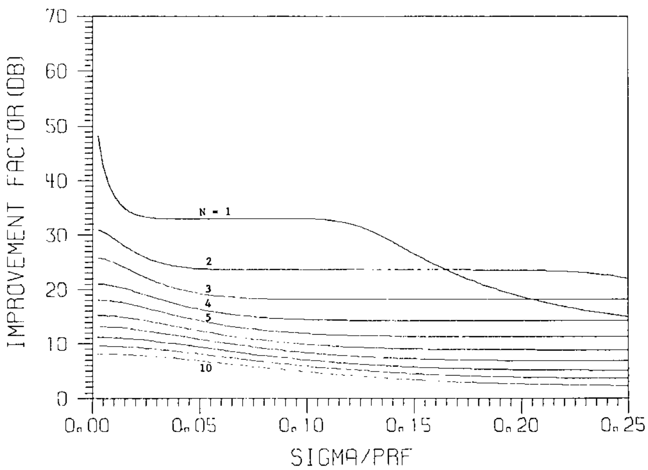


Fig. 8 — Improvement factor of surface clutter of a 16-point FFT doppler filter when pulses transmitted prior to the first MTI pulse are at a different frequency

range bins can be resolved. This is a typical residue-number problem. To find the natural number corresponding to a set of residue numbers a_1, a_2, ..., a_k, one may use the formula [3]

$$a_1 A_1 \frac{R}{r_1} + \ldots + a_k A_k \frac{R}{r_k} = S \bmod R, \tag{22}$$

where

$$A_i \frac{R}{r_i} = 1 \bmod r_i \tag{23}$$

and where the modular relation is defined as

$$A = a \bmod b, \tag{24a}$$

meaning

$$A = a + \ell b, \tag{24b}$$

where ℓ is an integer number.

As an example, if

$$r_1 = 2, r_2 = 3, r_3 = 5, r_4 = 7,$$

then

$$R = 2 \times 3 \times 5 \times 7 = 210,$$

$$\frac{R}{r_1} = 105,$$

$$\frac{R}{r_2} = 70,$$

$$\frac{R}{r_3} = 42,$$

$$\frac{R}{r_4} = 30.$$

According to Eq. (23)

$$105A_1 = 1 \bmod 2 \text{ or } A_1 = 1,$$

$$70A_2 = 1 \bmod 3 \text{ or } A_2 = 1,$$

$$42A_3 = 1 \bmod 5 \text{ or } A_3 = 3,$$

$$30A_4 = 1 \bmod 7 \text{ or } A_4 = 4.$$

By use of Eq. (22)

$$105a_1 + 70a_2 + 126a_3 + 240a_4 = S \bmod 210$$

If a_1, a_2, a_3, a_4 equals 1, 2, 0, 4, then

$$S = 95.$$

This algorithm can be easily implemented on a computer.

The method applies only to the case of a single target in one sweep. If there are more targets than one, the problem becomes much more complicated. For a doppler filter this probably is not a serious problem, since, if two targets have different doppler frequencies, they will fall into different doppler cells. This added information may be adequate to resolve multiple-target ambiguities. Two targets which have identical doppler frequencies are rare. For MTI, because no additional velocity resolution is available, resolution of range ambiguities becomes difficult. For example, if two targets fall into different range bins for each of n PRFs, there are 2^{n-1} possible range solutions. There is no known method to resolve this ambiguity.

CONCLUDING REMARKS

In this report we have shown that when a range-ambiguous MTI (or doppler filter) system has a shorter sampling time, the clutter correlation time is reduced and hence the clutter output becomes smaller. This assumes that the clutter is uniformly distributed. On the other hand, because of clutter foldover, the apparent clutter in each range bin is increased. The net change in improvement factor is a function of these two effects. For a doppler filter system, the improvement factor of a range-ambiguous system is worse than that of a range-unambiguous system. Furthermore range-ambiguity resolution in an MTI system for a multiple target situation becomes extremely difficult.

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